Universität Duisburg-Essen
Ingenieurwissenschaften / Informatik
Lecturer: Harsh Beohar

SS 2017
Sheet 2
15. May 2017

Exercise: "Modelling of concurrent systems"

The tutorial will take place on Monday (29.05.17) from 12:15–13:45 in Room LK 053. Please submit your solutions either in person to me OR email me by Wednesday (24.05.16), 1400 hours. Please note that late submissions (i.e., after 1400 hours) will not be accepted.

Task 1 *Comparitive concurrency semantics*

Prove the following facts (**(Points: 2+3)**):

(a) If two states of a transition system are bisimilar then they are failure equivalent.

(b) Show that finitary trace equivalence coincides with bisimulation under the assumption of determinism.

Task 2 *Hennessy-Milner Logic (HML)*

(a) Consider the following transition systems from the state s, t as depicted below: Are the two states s and t bisimilar? If yes, argue why they satisfy the same set

$$\begin{tikzcd} s \arrow[r, "a"] & s_1 \arrow[r, "b"] & s_2 \arrow[ll, bend right, "a"'] \arrow[loop right, "b"] \end{tikzcd}$$

$$\begin{tikzcd} t \arrow[r, "a"] & t_1 \arrow[r, "b"] \arrow[loop below, "b"] & t_2 \arrow[ll, bend right, "a"'] \end{tikzcd}$$

of HML formulas. Otherwise give a formula expressible in HML that distinguishes the two upto bisimilarity. **(Points: 1+1)**

(b) Recall the Hennessy-Milner logic and define the following two operations:

i) Define an operation on arbitrary (finite) subsets of actions $X \subseteq L$ as follows:

$$\tilde{X} = \begin{cases} \bigwedge_{a \in X}[a]0 & \text{if } X \neq \emptyset \\ 1 & \text{if } X = \emptyset. \end{cases}.$$

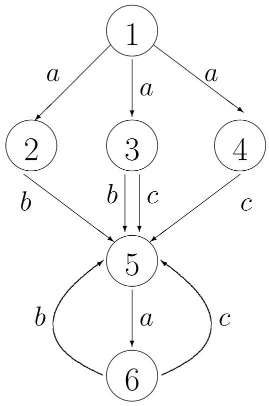
The notation $\bigwedge$ denotes the generalised AND operation, for instance, if $X = \{b, c\}$, then $\bigwedge_{a \in X} = [b]0 \wedge [c]0$. Note that here 0 and 1 denotes the logical constants *false* and *true*, respectively.

ii) Extend the diamond modality to a nonempty sequence of actions $w \in L^* \setminus \{\epsilon\}$ as follows: $\langle\langle w \rangle\rangle = \begin{cases} \langle a \rangle & \text{if } w = a \\ \langle\langle w' \rangle\rangle \langle\langle a \rangle\rangle & \text{if } w = w'a \wedge w' \neq \epsilon. \end{cases}$

As usual, let $(S, L, \rightarrow)$ be an labelled transition system space. Show that if (w, X) is a failure pair of the state s (i.e., $(w, X) \in \mathcal{F}(s)$) with w being a nonempty word, then the state s satisfies the formula $\langle\langle w \rangle\rangle \tilde{X}$, i.e., $s \models \langle\langle w \rangle\rangle \tilde{X}$. **(Points: 4)**

Task 3 *Computing bisimular pairs*

Using the bisimulation algorithm described in the lecture, compute the bisimilar state pairs of the following transition system. **(Points: 4)**



Task 4 *Elementary algebra*

Recall the theory T of Peano arithmetic as presented in the lecture whose signature Σ consists of a constant $\mathbf{0}$, a unary operation $\mathbf{s}$, and two binary operations $\mathbf{a}, \mathbf{m}$ subject to the following axioms.

$$\mathbf{a}(x, \mathbf{0}) = x \tag{1}$$
$$\mathbf{a}(x, \mathbf{s}(y)) = \mathbf{s}(\mathbf{a}(x, y)) \tag{2}$$
$$\mathbf{m}(x, \mathbf{0}) = \mathbf{0} \tag{3}$$
$$\mathbf{m}(x, \mathbf{s}(y)) = \mathbf{a}(\mathbf{m}(x, y), x) \tag{4}$$

(a) Consider a simplification T' of T in which the binary operations $\mathbf{a}, \mathbf{m}$ are disallowed. Then, how many number of terms can be generated in T' when the set of variables is an empty set? **(Points: 1)**

(b) Give a derivation of the fact that $T \vdash \mathbf{m}(\mathbf{s}(0), \mathbf{s}(0)) = \mathbf{s}(0)$. **(Points: 3)**

(c) Consider the algebra of Booleans $\mathbb{B} = (\{0, 1\}, \wedge, \oplus, \neg, 0)$, where $0, 1$ are the logical constants false and true, $\wedge$ is the Boolean *AND* operation, $\neg$ is the logical negation, and $\oplus$ is the binary *exclusive-or* operation on Booleans. Consider the following interpretation $\iota : \{\mathbf{0}, \mathbf{s}, \mathbf{a}, \mathbf{m}\} \to \{0, 1, \wedge, \oplus, \neg\}$ defined as:

$$\iota(\mathbf{0}) = 0,\ \iota(\mathbf{s}) = \neg,\ \iota(\mathbf{a}) = \oplus,\ \text{and}\ \iota(\mathbf{m}) = \wedge\ .$$

Argue whether $\mathbb{B}, \iota \models T$ holds or not. **(Points: 4)**

Task 5 *Order induced by summands*

Recall the process algebra BSP and consider the following relation $\leq$ on the open terms of BSP:

$$x \leq y \iff \mathrm{BSP}(A) \vdash x + y = y.$$

(a) Prove that the relation $\leq$ is a partial order. **(Points: 4)**

(b) Give an example of two closed BSP(A) terms p, q such that $p \leq q$ but not $a.p \leq a.q$ (with $a \in A$). **(Points: 3)**