Universität Duisburg-Essen **SS 2016**
Ingenieurwissenschaften / Informatik **Sheet 3**
Lecturer: Harsh Beohar **19. June 2017**

Exercise: "Modelling of concurrent systems"

The tutorial will take place on Friday (30.06.17) from 17:15–18:45 in Room LF 265. Please submit your solutions either in person to me OR email me by Thursday (29.06.17), 1700 hours. Please note that late submissions (i.e., after 1700 hours) will not be accepted.

Task 1 *Order induced by summands*

Recall the process algebra BSP and consider the following relation $\leq$ on the open terms of BSP:

$$x \leq y \iff \mathrm{BSP}(A) \vdash x + y = y.$$

(a) Prove that the relation $\leq$ is a partial order. **(Points: 4)**

(b) Give an example of two closed BSP(A) terms p, q such that $p \leq q$ but not $a.p \leq a.q$ (with $a \in A$). **(Points: 2)**

Task 2 *Basic sequential processes*

Prove the following two statements.

(a) Let p, q, r be closed BSP(A)-terms. If $(p+q)+r \leftrightarrow r$, then $p+r \leftrightarrow r$ and $q+r \leftrightarrow r$. **(Points: 2)**

(b) Show that there is no closed BSP(A)-term p such that $p \leftrightarrow a.p$. **(Points: 4)**

Task 3 *Recursion I: On syntax*

Recall the quotient term algebra $\mathbb{P}(\mathrm{BSP}(A))/\leftrightarrow$ from the lectures.

(a) Find two different solutions of the recursive equation $X = X + a.0$ in the term model $\mathbb{P}(\mathrm{BSP(A)})/\leftrightarrow$. **(Points: 2)**

(b) Determine whether the following recursive specifications are guarded or unguarded: $\{X = Y, Y = a.X\}$ and $\{X = a.Y + Z, Y = b.Z + X, Z = c.X + Y\}$. **(Points: 2)**

(c) Assume that the set of actions A is empty, i.e., $A = \emptyset$. Then, give an unguarded recursive specification with only one solution in the term model $\mathbb{P}(\mathrm{BSP(A)})/\leftrightarrow$. **(Points: 2)**

Task 4 *Recursion II: On semantics*

(a) Give the transition system induced by the following process terms: $\mu(a.X).\{X = a.b.X + b.Y + 0, Y = X + a.(Y + X)\}$ and $\mu(b.X + 1).\{X = a.X + Y, Y = a.Y\}$. **(Points: 2)**

(b) Consider the recursive specification $E = \{X = a.X + b.Y, Y = a.Y + b.X\}$. Can we derive $(\mathrm{BSP} + E + \mathrm{RSP})(A) \vdash X = Y$? If not, construct a recursive specification E' from which it follows that $(\mathrm{BSP} + E + E' + \mathrm{RSP})(A) \vdash X = Y$? In any case, please support your answer formally. **(Points: 5)**