Universität Duisburg-Essen
Ingenieurwissenschaften / Informatik
Lecturer: Harsh Beohar

SS 2016
Sheet 3
3. July 2017

Exercise: "Modelling of concurrent systems"

The tutorial will take place on Friday (14.07.17) from 1700 in Room LF 265. Please submit your solutions either in person to me OR email me by Thursday (13.07.16), 1700 hours. Please note that late submissions (i.e., after 1700) will not be accepted.

Task 1 *Elimination of sequential composition*

Prove the following two statements.

(a) Let p and q be any two closed BSP(A) terms. Show that there exists a closed BSP(A) term r such that TSP(A) $\vdash p \cdot q = r$. **(Points: 3)** [Hint: Use structural induction!]

(b) Use the above result to prove the following *elimination* result: For any closed TSP(A) term p, there exists a closed BSP(A) term q such that TSP(A) $\vdash p = q$. **(Points: 3)**

Task 2 *More on sequential behaviour*

Prove the following statements.

(a) Without using the axiom $x \cdot (y \cdot z) = (x \cdot y) \cdot z$, prove that for any three closed TSP(A)-terms, p, q, and r, we have $(p \cdot q) \cdot r = p \cdot (q \cdot r)$. **(Points: 3)**

(b) Show that the axiom $(x + y) \cdot z = x \cdot z + y \cdot z$ of the theory TSP is sound up-to bisimulation. **(Points: 2)**

Task 3 *Parallel communicating processes*

(a) Assume that there is no communication, i.e., $\gamma = \emptyset$. Argue whether the following equations hold in BCP($A, \emptyset$). **(Points: 3)**

i) $a.b.1 \parallel c.1 = a.b.c.1 + a.c.b.1 + c.a.b.1$;

ii) $(a.1 + b.1) \parallel c.1 = a.c.1 + b.c.1 + c.(a.1 + b.1)$;

iii) $a.1 \parallel 0 = a.0$

(b) Let $\gamma(a, b) = c$ be the only defined communication and let $H = \{a, b\}$. Show that $\partial_H(c.(a.1 + b.1) \parallel b.1)$ is deadlock free, where a closed BCP(A, γ)-term is deadlock free iff it is derivably equal to a closed BSP(A)-term without 0 occurrence. Also, show that $\partial_H((c.a.1 + c.b.1) \parallel b.1)$ does have a deadlock. **(Points: 2)**

(c) Show that for arbitrary BCP($A, \emptyset$)-terms x, y, we have if $x = x + 1$ and $y = y + 1$, then BCP+FMA($A, \emptyset$) $\vdash x|y = 1$. **(Points: 2)**

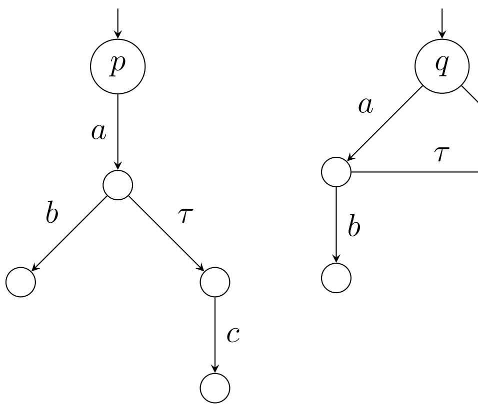
(d) Consider the following family of operators (for each $B \subseteq A$) whose semantics is defined by the following rules:

$$\frac{x \xrightarrow{a} x' \quad a \notin B}{x|[B]|y \xrightarrow{a} x'|[B]|y} \quad \frac{y \xrightarrow{a} y' \quad a \notin B}{x|[B]|y \xrightarrow{a} x|[B]|y'} \quad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{a} y' \quad a \in B}{x|[B]|y \xrightarrow{a} x'|[B]|y'}.$$

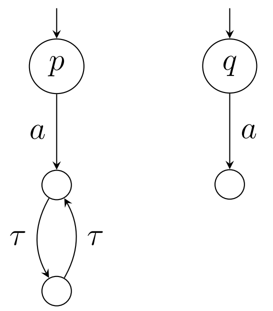
Define this family of operators $\_|[B]|\_$ using only the parallel composition operator, the encapsulation operator, and the renaming operator? Please explain your construction formally. **(Points: 4)**

Task 4 *(Rooted) branching bisimulation*

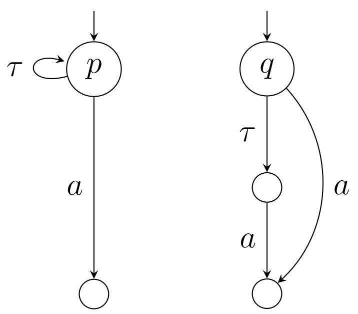
Consider the following pairs of transition systems with the initial states p, q. In each case, argue whether the two transition systems are (rooted) branching bisimilar. Support your answer either by giving the witnessing relation explicitly or explain why they cannot be related by a (rooted) branching bisimulation relation. **(Points: 8)**



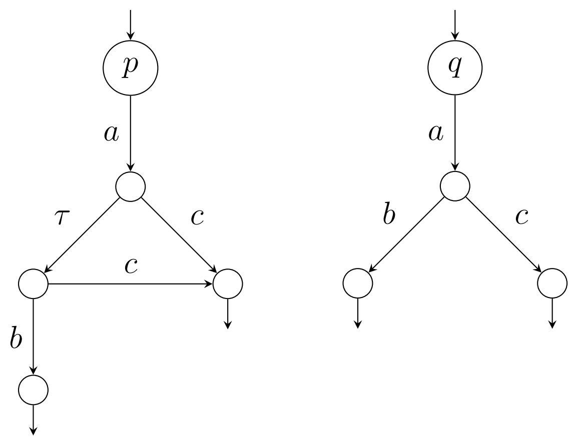
(a) Pair 1



(b) Pair 2



(c) Pair 3



(d) Pair 4