Universität Duisburg-Essen **SS 2016**
Ingenieurwissenschaften / Informatik **Sheet 5**
Lecturer: Harsh Beohar **30. June 2016**

Exercise: "Modelling of concurrent systems"

The tutorial will take place on Thursday (27.07.17) from 10:15–11:45 in Room LB 117. Please submit your solutions either in person to me OR email me by Wednesday (26.07.17), 1200 hours. Please note that late submissions (i.e., after 1200 hours) will not be accepted.

Task 1 *Weak bisimulation*

Let $(S, A_\tau, \rightarrow)$ be a labelled transition system space. A binary relation $R \subseteq S \times S$ is a *weak bisimulation* relation if and only if the following transfer properties are satisfied.

(a) $\forall_{s,t,s',\in S, a \in A_\tau} \left((s \xrightarrow{a} s' \wedge sRt) \implies \exists_{t' \in S} \left(t \overset{\hat{a}}{\twoheadrightarrow} t' \wedge s'Rt'\right)\right);$

(b) Symmetric with the roles of s and t interchanged.

Two states are *weakly bisimilar*, denoted $s \leftrightarrows_w t$, if there exists a weak bisimulation relation R such that sRt.
Note that, in the above definition, $\twoheadrightarrow \subseteq S \times A^* \times S$ is the weak reachability relation as defined in the lectures and the notation $s \overset{\hat{a}}{\twoheadrightarrow} t$ means $s \overset{a}{\twoheadrightarrow} t$ or $s = t$ and $a = \tau$.

(a) Show that if R and R' are two weak bisimulation relations then their relation composition $R \circ R'$ is also a weak bisimulation relation. **(Points: 3)**

(b) Given an example of two states that are weakly bisimilar but not branching bisimilar. **(Points: 3)**

Task 2 *Branching axiom*

(a) For any arbitrary $\mathrm{TCP}_\tau(A, \gamma)$-terms x, y, z, show that

$$\mathrm{TCP}_\tau(A,\gamma) \vdash x \lfloor\!\lfloor (\tau.(y+z)+y) = x \lfloor\!\lfloor (y+z). \qquad \textbf{(Points: 2)}$$

(b) For any arbitrary $\mathrm{TCP}_\tau(A, \gamma)$-terms x, y, derive the branching axiom $a.(\tau.(x+y)+x) = a.(x+y)$ using the above equation (i.e., $x \lfloor\!\lfloor (\tau.(y+z)+y) = x \lfloor\!\lfloor (y+z)$) and the other remaining axioms of $\mathrm{TCP}_\tau(A, \gamma)$. **(Points: 2)**

Task 3 *Two one-place buffer in parallel is a two-place buffer*

Let D be a finite set of data. Consider the following recursive specifications

(a) A one place buffer with input channel i and output channel l.

$$B1_{il} = 1 + \sum_{d \in D} i?d.l!d.B1_{il} \ .$$

(b) A two-place buffer with input channel i and output channel o.

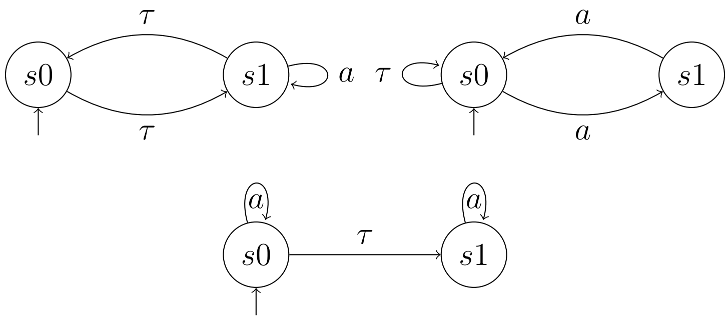
$$
\begin{aligned}
B2_{io} &= \; 1 + \sum_{d \in D} i?d.B_d \ , \\
B_d &= \; o!d.B2_{io} + \sum_{e \in D} i?e.o!d.B_e \ , \qquad \text{for every } d \in D.
\end{aligned}
$$

Assume standard communication, i.e., $\gamma(l?d, l!d) = \gamma(l!d, l?d) = l{!?}d$, for every $d \in D$, and undefined otherwise. Show that $\mathrm{TCP}_{\tau,\mathrm{rec}} + \mathrm{RSP} \vdash B2_{io} = \tau_I(\partial_H(B1_{il} \parallel B1_{lo}))$, where $H = \{l?d, l!d \mid d \in D\}$ and $I = \{l{!?}d \mid d \in D\}$. **(Points: 5)**

<u>Task 4</u> *Confluence*

(a) Show that all τ-confluent steps in a given transition system are inert modulo $\leftrightarrow_b$. Does the converse also holds? Argue your answer formally. **(Points: 2)**

(b) Give the maximal τ-confluent sets for the following three transition systems. Also, inform in which cases the τ-prioritisation theorem holds and give the corresponding reduced systems. **(Points: 3)**



<u>Task 5</u> *Probabilistic process algebra*

Consider the terms $p \equiv 1 \oplus_{\frac{1}{2}} 0$ and $q \equiv (1 \oplus_{\frac{1}{3}} (1 + 1)) \oplus_{\frac{1}{2}} 0$ in the theory $\mathrm{BSPprb}(A)$. Give the transition systems induced by the terms p and q. Argue whether the terms p and q are probabilistically bisimilar? **(Points: 5)**