Course "Modelling of Concurrent Systems"
Summer Semester 2017
University of Duisburg-Essen

Harsh Beohar
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Course handler

Harsh Beohar

- Room LF 265
- E-Mail: harsh.beohar@uni-due.de
- Meeting by appointment.
- Please send mail only by your official student mail id's.

http://www.uni-due.de/zim/services/e-mail/

Task: Lecturer + Exercise Tutor.

Web-Seite:

http://www.ti.inf.uni-due.de/teaching/ss2017/mod-ns/

Lecture schedule

Schedule:

- Monday, 12:15-13:45, in Room LK 053
- Thursday, 10:15-11:45, in Room LB 117

Exercises

Schedule:

(Roughly, every fourth lecture kept for exercises modulo holidays.)

- Thursday, 11/05, 10:15-11:45, in Room LB 117.
- Monday, 22/05, 12:15-13:45, in Room LK 053.
- Monday, 12/06, 12:15-13:45, in Room LK 053.
- Monday, 03/07, 12:15-13:45, in Room LK 053.
- Monday, 17/07, 12:15-13:45, in Room LK 053.
- Thursday, 27/07, 10:15-11:45, in Room LB 117.

Idea:

- Problem sheet will be announced in the class, whenever it is published.
- At the same time, also the deadline to submit the exercises will also be announced.
- Please hand in your solutions at the start of the lecture.

Exercises

Scheme:

- If the sum is more than 60% and once a solution is presented on board then you get a bonus point.
- Effect is improvement by one grade level. E.g. 2.3 to 2.0
- Group solutions are not allowed.

Target audience

MAI

Master "Applied computer science" ("Angewandte Informatik") - focus engineering or media computer science:

- In the brochure you can find the field of application:
  "Distributed Reliable Systems" ("Verteilte, Verlässliche Systeme")
  Concurrent systems (Nebenläufige Systeme)

Stundenzahl: *4 SWS (3V + 1Ü), 6 Credits*

Target audience

Master ISE/CE – Verteilte, Verlässliche Systeme

In Master "ISE – Computer Engineering", this lecture is classified as follows:

- Elective "Verteilte, Verlässliche Systeme"
  (Reliable Systems)
  Stundenzahl: *4 SWS (3V + 1Ü)*

Requirement

Prerequisites:

- Automata and Formal languages.

For the past teaching content, see (although addition of event structures is new)
`http://www.ti.inf.uni-due.de/teaching/ss2016/mod-ns/`

Examination

The exam will be held as a viva voce (oral examination).
Planned dates:
14th August 2017 (Monday) and 15th August 2017 (Tuesday).

Please enroll yourself at the Secretariat in the room LF 227.

Literature

- Jos Baeten, Twan Basten, and Michel Reniers.

  Process algebra: Equational theories of communicating processes Cambridge University Press, 2010.

  Contents: **(Probabilistic) Process algebra**.
- Luca Aceto, Anna Ingólfsdóttir, Kim G. Larsen, Jiri Srba.

  Reactive Systems: Modelling, Specification and Verification. Cambridge University Press, 2007.

  Contents: **Strong and weak bisimulation, Hennessy-Milner logic, Timed automata**
- Glynn Winskel.

  Event structures. Invited lectures for the Advanced Course on Petri Nets, Sept. 1986. Appears as a report of the Computer Laboratory, University of Cambridge, 1986. https://www.cl.cam.ac.uk/ gw104/EvStr.pdf

  Contents: **Event structures**

Literature in Process algebra

- Robin Milner.

  Communication and Concurrency. Prentice Hall, 1989.

  Contents: **Process calculus (CCS), Strong and weak bisimulation, Hennessy-Milner logic.**

- Tony Hoare.

  Communicating sequential processes 2004. Available at `http://www.usingcsp.com/cspbook.pdf`

  Contents: **Process calculus CSP, Failure equivalence**

- Bill Roscoe.

  The Theory and Practice of Concurrency 1997. Available at `http://www.cs.ox.ac.uk/people/bill.roscoe/publications/68b.pdf`.

  Contents: **A reference book on CSP**

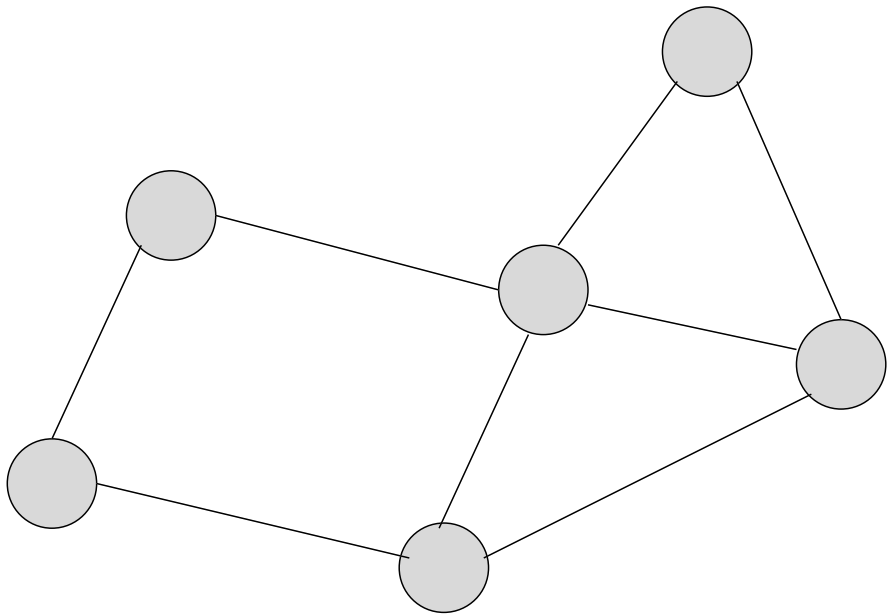
Lecture style

- We will follow "the" process algebra book. Also, which sections are to be read for the next lecture will be announced in the current one.
- Very few materials will be presented using slides and mostly on blackboard. So please make your own notes!

Motivation

What are concurrent systems?

In general: systems in which several components/processes run concurrently and typically communicate via message passing.



Motivation

Concurrency versus parallelism:

Motivation

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Parallelism

Two events take place in parallel if they are executed at the same moment in time.

Concurrency

Two events are concurrent if they *could potentially* be executed in parallel, but they do not have to. This means there is no causal dependency between them.

Motivation

Concurrency versus parallelism:

Parallelism

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Concurrency

Two events are concurrent if they *could potentially* be executed in parallel, but they do not have to. This means there is no causal dependency between them.

Hence: concurrency is the more general term.
Examples?

Motivation

(Potential) characteristics of concurrent systems

- Concurrency/parallelism
- Openness (extendability, interaction with the environment)
- Modularity
- Non-terminating behaviour (infinite runs)
- Non-determinism
- Temporal properties (e.g. "an event will occur eventually")

Motivation

Problems with concurrent systems

- Deadlocks
- Guaranteeing mutual exclusion
- Infinite respectively huge state space
- Strongly dynamic behaviour/changing number of processes
- Variable topology/mobility

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Hence: We need methods to model, analyze and verify such systems.

Change in view

Classic view

- Program is a function that transform an input into output.
- Two programs are equivalent if and only if they compute the same output for every input.

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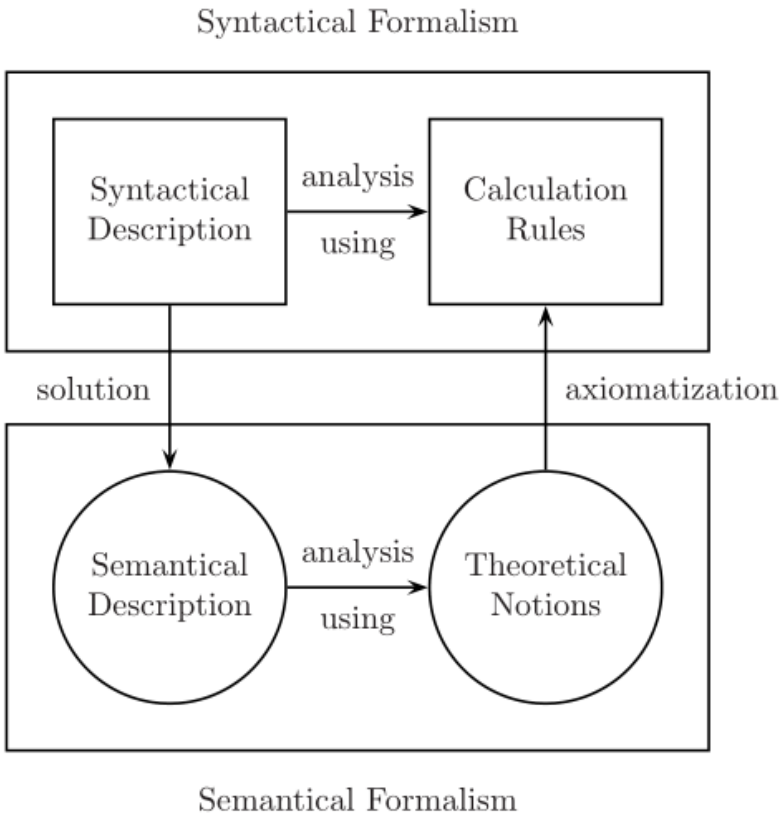
Mathematical modelling

- Inspired from traditional engineering disciplines.
- Make system models in formal way.
- Analyse them.
- Then build the 'real' system and test it against those models.

Mathematical modelling (Cuijpers 2004.)

Syntax aims at concise and finite way of handling semantical notions, which can be infinite mathematical objects.



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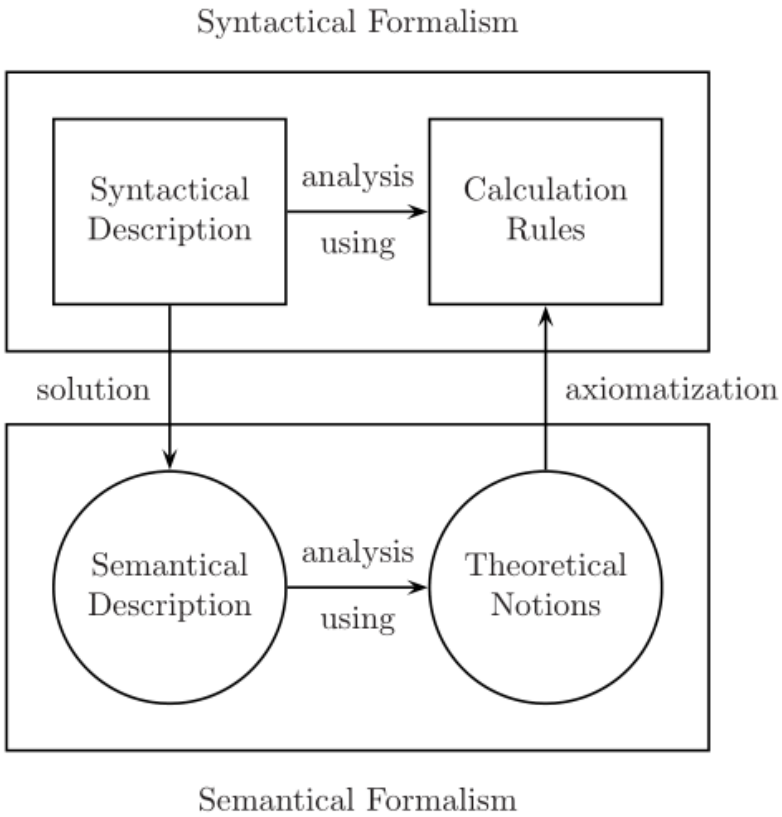

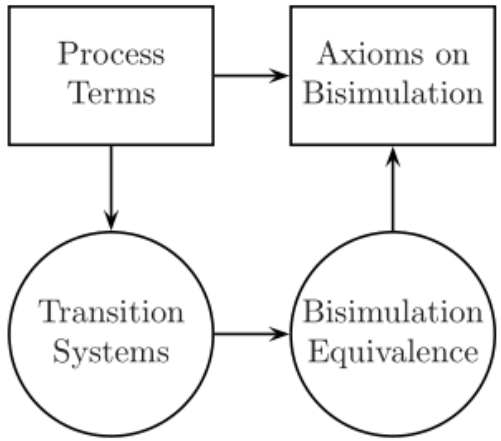

Computer Science

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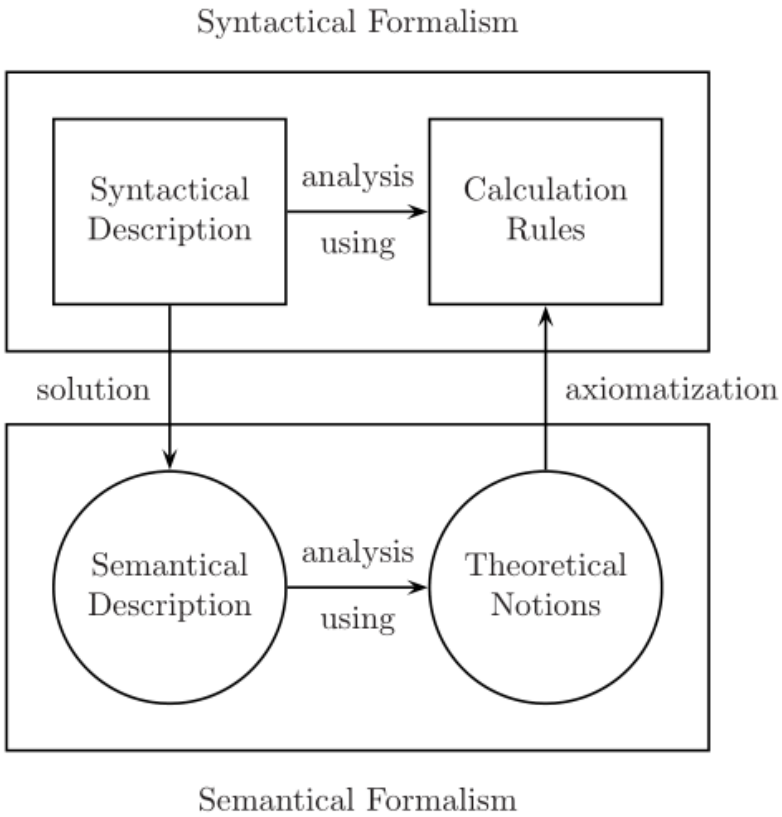
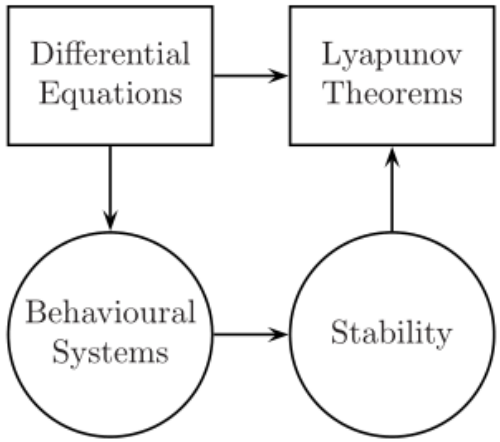




Control Theory

Table of contents

We will introduce the following models for concurrent systems:

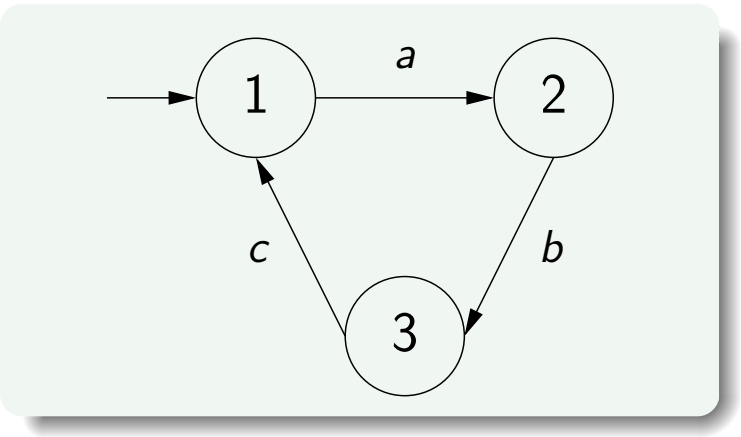
- Transition systems
- Models which are closer to realistic programming languages (for instance process calculi)
- Additional models: Event structure

Furthermore (in order to investigate/analyze systems):

- Specification of properties of concurrent systems (Hennessy-Milner logics)
- Behavioural equivalences: When do two systems behave the same (from the point of view of an external observer)?

Transition systems

- Transition systems represent states and transitions between states.
- True parallelism is not directly represented.
- Strong similarity to automata, however we are here not so much interested in the accepted language.



Reviews of some notions

Definitions

- For a set X, we write X^* for the set of all finite words including the empty one ε.
- For a set X, we write X^ω the set of all infinite words. Also, we write $X^\infty = X^* \cup X^\omega$.
- A binary relation R between the sets X and Y is a subset of $X \times Y$, i.e., $R \subseteq X \times Y$.
- Often, we write xRy iff $(x, y) \in R$.
- A preorder $R \subseteq X \times Y$ is a

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- A partial order (poset) R is a preorder that is antisymmetric.
- An equivalence relation R is a partial order that is symmetric.

Transition system space

Formal definition

A transition system space is a triple $(S, L, \rightarrow)$ of

1. a set of states S;
2. a set of labels L;
3. a transition relation $\rightarrow \subseteq S \times L \times S$;

Notations:

- $s \xrightarrow{a} t \iff (s, a, t) \in \rightarrow$
- $s \xrightarrow{a} \iff \nexists_{t \in S}\ s \xrightarrow{a} t$

Basics

Example on board.

Definition

The reachability relation $\rightarrow^* \subseteq S \times L^* \times S$ is inductively defined as follows:

$$\frac{}{s \overset{\varepsilon}{\twoheadrightarrow} s} \qquad \frac{s \overset{w}{\twoheadrightarrow} s' \quad s' \overset{a}{\rightarrow} s''}{s \overset{wa}{\twoheadrightarrow} s''}$$

The transition system induced by state s consists of all states reachable from s, and it has the transitions and final states induced by the transition system space.

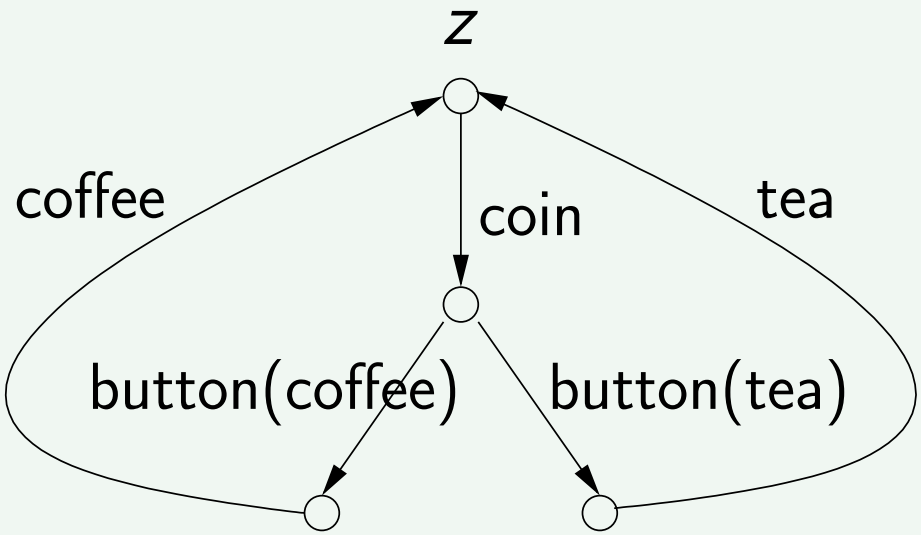
Transition systems (examples)

A classical example: the tea/coffee-machine

We want to model a very simple machine that

- outputs tea or coffee after a coin has been inserted and a button has been pressed,
- can show faulty behaviour *and*
- may potentially behave non-deterministically.

Transition systems (examples)



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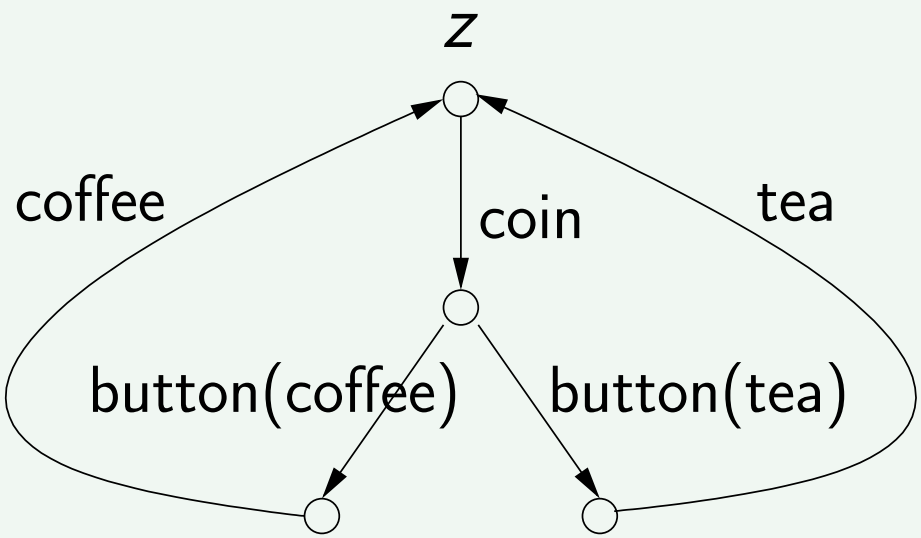

A tea/coffee-machine.

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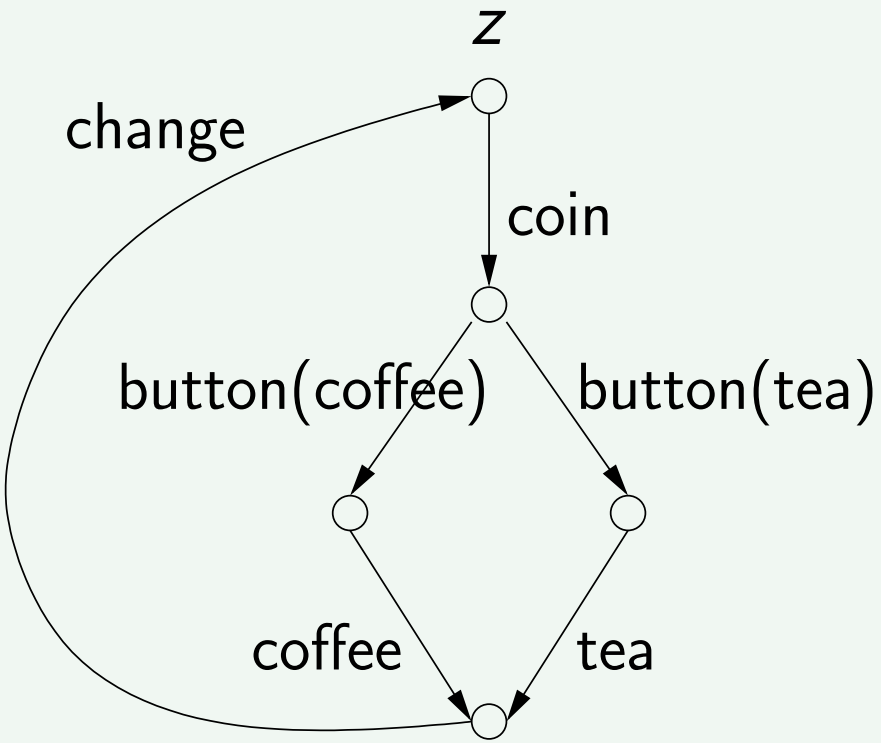

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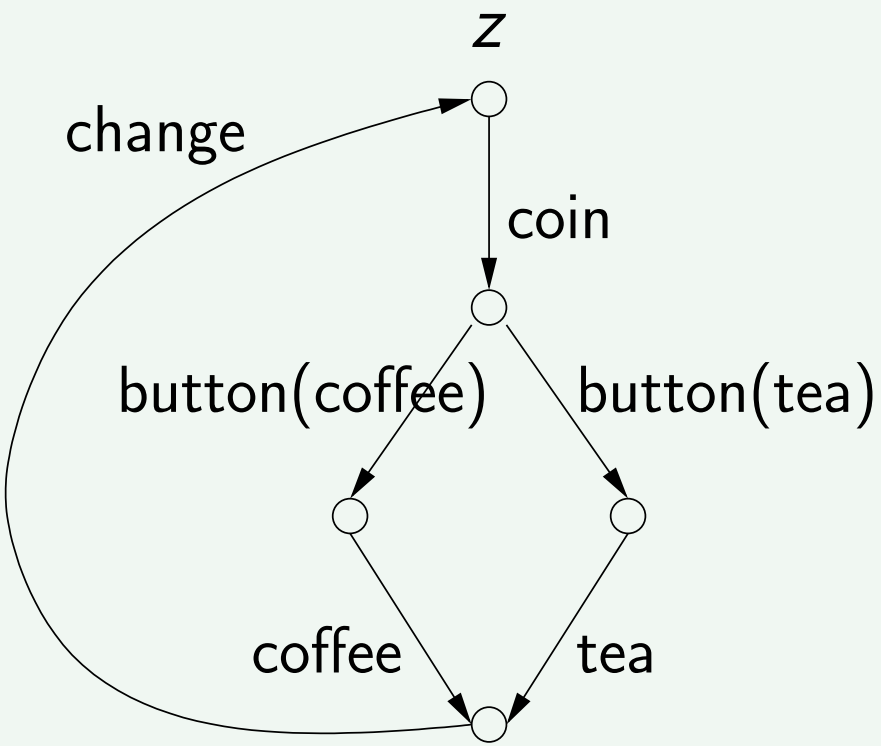
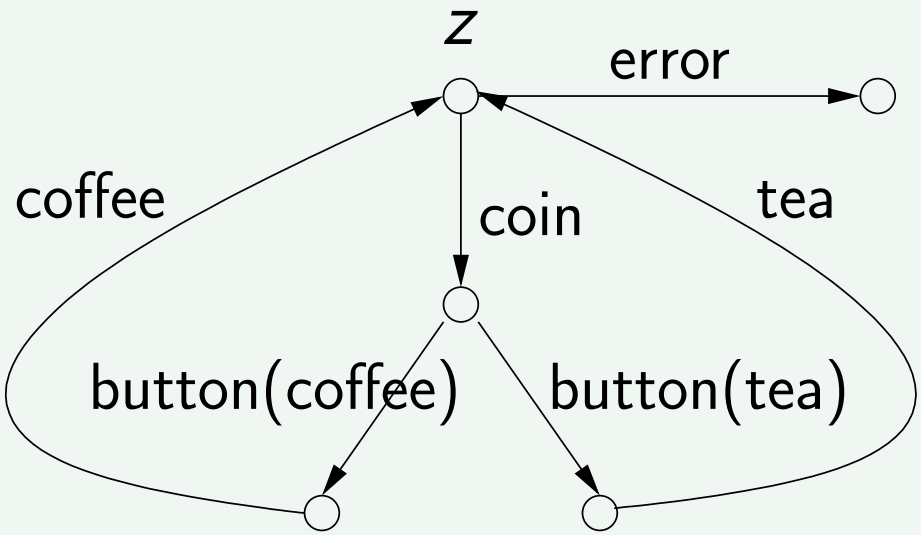


A machine that gives back change.

Transition systems (examples)



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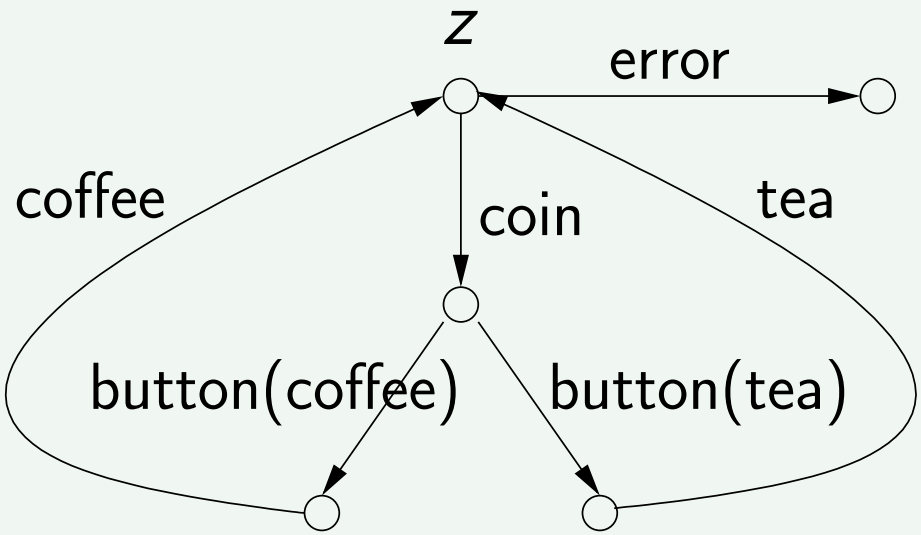
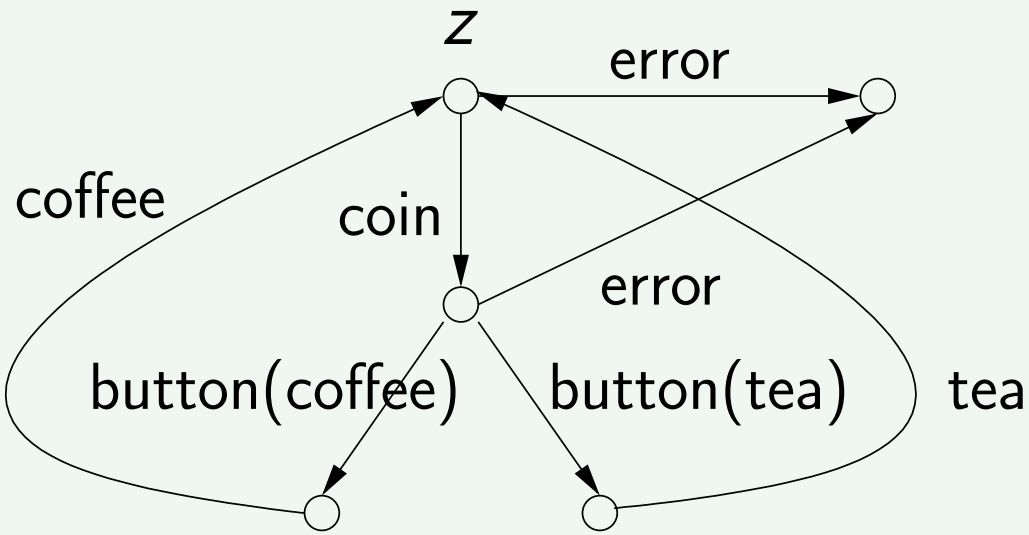


A machine with an error. The occurrence of an error is actually rather an internal action and could alternatively be modelled with a τ.

Transition systems (examples)



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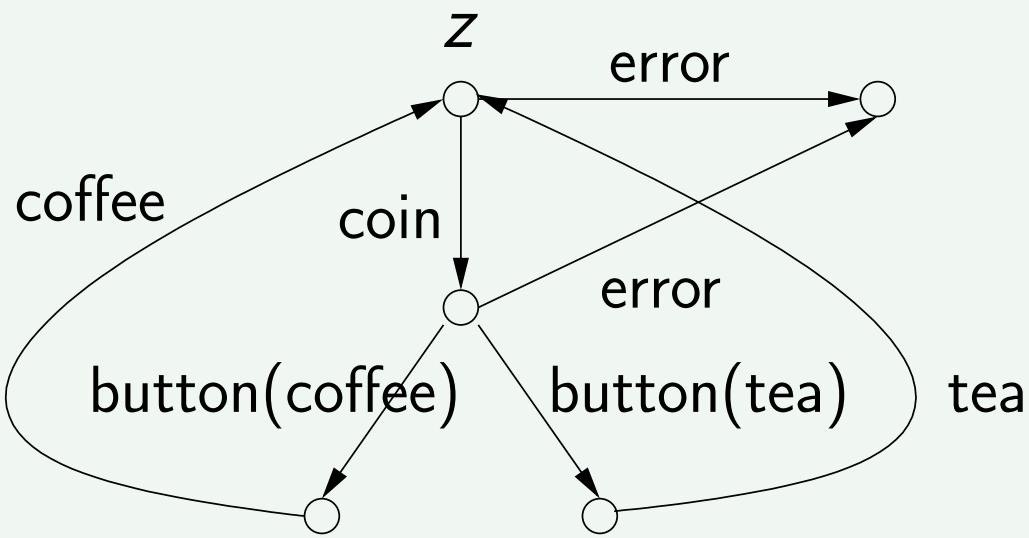
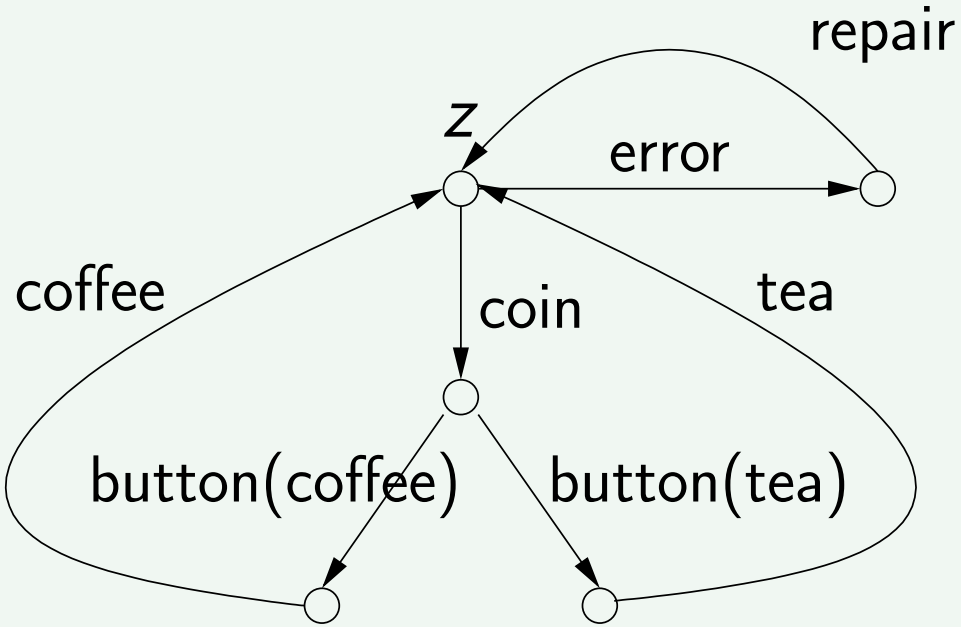


An (unfair) machine with faulty behaviour which may enter the error state after a coin has been inserted.

Transition systems (examples)



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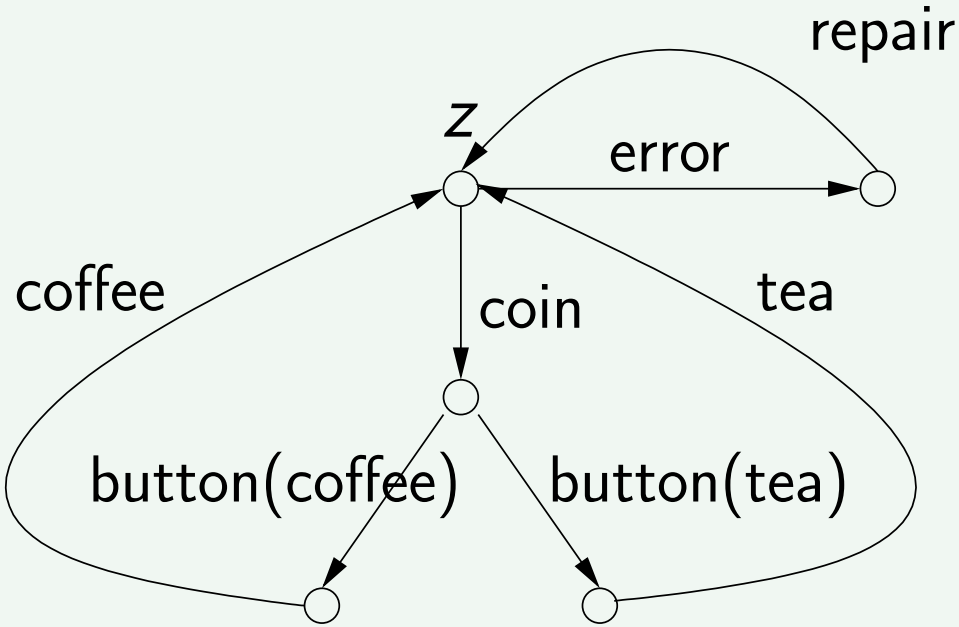
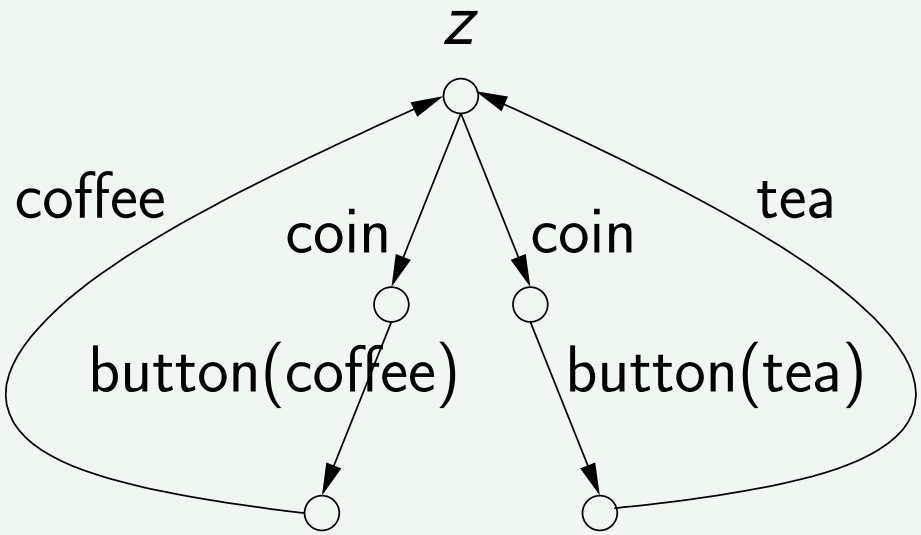


A machine with an error state that can be repaired.

Transition systems (examples)



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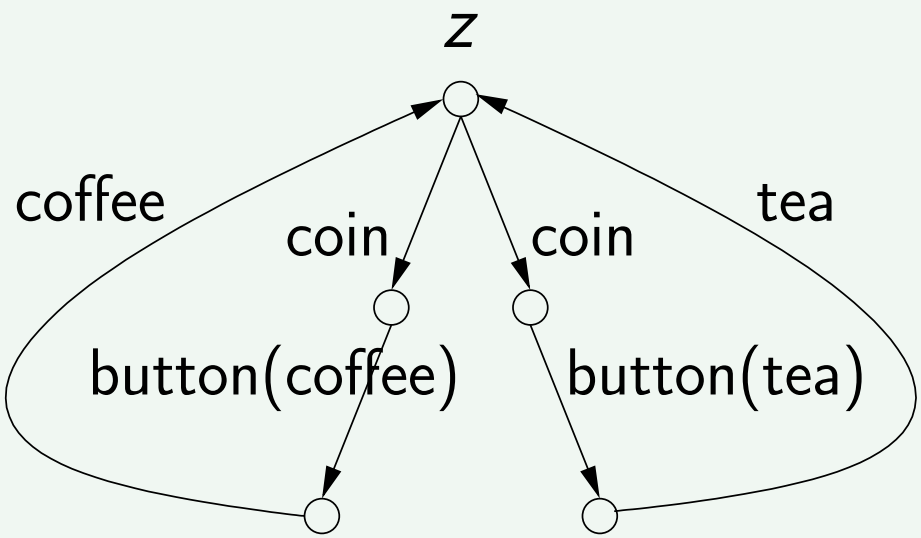

A machine with non-deterministic behaviour that makes a choice of beverages for the user.

Deterministic transition systems

Deterministic transition system (definition)

A state s of a transition system is *deterministic*

$$\forall_{s',s'' \in S, a \in L} \left(s \xrightarrow{a} s' \wedge s \xrightarrow{a} s'' \right) \implies s' = s''$$

A transition system induced by state s is deterministic if every reachable states from s is deterministic.

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A transition system induced by state s is deterministic if every reachable states from s is deterministic.

Remarks:

- All tea/coffee-machines, apart from the last, are deterministic.
- Opposed to deterministic finite automata we do not require for deterministic transition systems that every action is feasible in every state.

Some more definitions

- A state s of a transition system is a *deadlock* state iff

$$\forall_{a \in L} \nexists_t \; s \xrightarrow{a} t.$$

  A transition system starting from s has a deadlock iff a deadlock state is reachable from s.
- A transition system is *regular* iff both its set of states and transitions are finite.
- A transition system is *image finite* iff each of its states has only finitely many outgoing transitions.

Behavioural equivalences (bisimilarity)

Similar to the minimization procedure for (deterministic) finite automata, there exists a method for determining bisimilar pairs of states in a transition system.

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Similar to the minimization procedure for (deterministic) finite automata, there exists a method for determining bisimilar pairs of states in a transition system.

Idea:

- Start with a very coarse relation $\sim_0$ that relates all possible states.
- Refine this relation step by step and construct relations $\sim_1$, $\sim_2$, ...
- As soon as two subsequent relations coincide ($\sim_n = \sim_{n+1}$) we have found the bisimilarity (at least for finite transition systems). That is, we have $\leftrightarrows = \sim_n$.

Behavioural equivalences (bisimilarity)

Method for determining bisimilar pairs of states

Input: A transition system $T = (S, L, \rightarrow)$

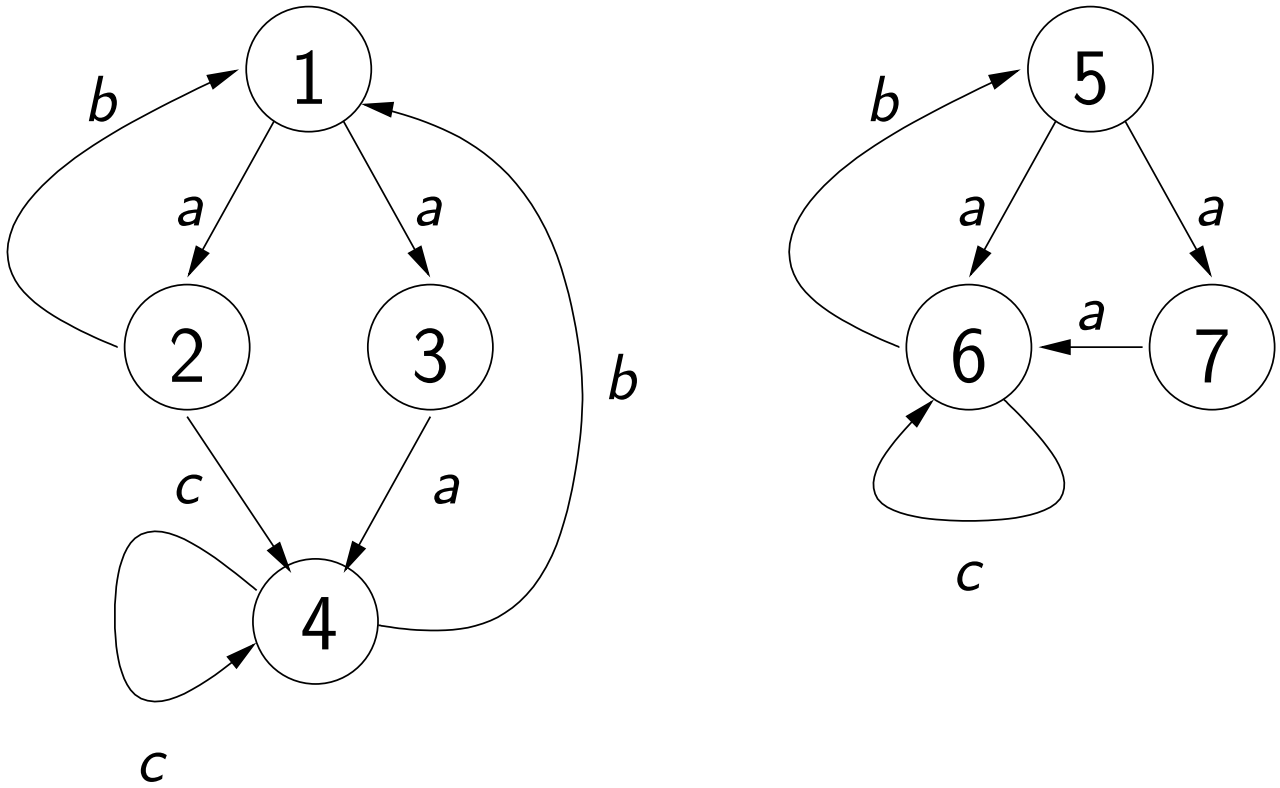
- Define $\sim_0 = S \times S$.
- $\sim_{n+1} \subseteq S \times S$, where $s \sim_{n+1} s'$ if and only if for all $a \in L$:
  1. For every t with $s \xrightarrow{a} t$ there exists t' such that $s' \xrightarrow{a} t'$ and $t \sim_n t'$.
  2. For every t' with $s' \xrightarrow{a} t'$ there exists t such that $s \xrightarrow{a} t$ and $t \sim_n t'$.

The method terminates as soon as $\sim_n = \sim_{n+1}$.

Output: $\sim_n$

Behavioural equivalences (bisimilarity)

Example: determine the bisimilar pairs of states of the following transition system



Behavioural equivalences (bisimilarity)

If we represent the equivalence relations $\sim_i$ via equivalence classes, then we obtain the following sequence $\sim_0, \sim_1, \sim_2 = \sim_3$.

$\sim_0$

1	3	2	6
5	7	4	

$\sim_1$

1	3	2	6
5	7	4	

$\sim_2$

1	3	2	6
5	7	4	

$=$

$\sim_3$

1	3	2	6
5	7	4	

Behavioural equivalences (bisimilarity)

Lemma

It holds that:

1. $\sim_n$ is an equivalence relation for all $n \in \mathbb{N}$.
2. $s \sim_n s'$ implies $s \sim_m s'$ for all $m \leq n$.
3. $s \leftrightarrows s'$ implies $s \sim_n s'$ for all $n \in \mathbb{N}$.
4. $\sim_n = \sim_{n+1}$ implies $\sim_n = \sim_m$ for all $m \geq n$.

Behavioural equivalences (bisimilarity)

Proposition

Let $T = (S, L, \rightarrow)$ be an image finite transition system space, i.e., for every state s the set

$$\{t \mid \exists a \in L : s \xrightarrow{a} t\}$$

is finite.

Then we have $s \leftrightarrows t$ if and only if $s \sim_n t$ for all $n \in \mathbb{N}$.

In other words: $\leftrightarrows = \bigcap_{n \in \mathbb{N}} \sim_n$.

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This proposition does not hold for transition systems which are not finitely branching.