Universität Duisburg-Essen
Ingenieurwissenschaften / Informatik
Lecturer: Harsh Beohar

SS 2017
Sheet 1
24. April 2017

Exercise: "Modelling of concurrent systems"

The tutorial will take place on Thursday (11.5.17) from 10:15–11:45 in Room LB 117.

Task 1 *Mutual exclusion*

Consider two processes P_1 and P_2 that share a common resource R. The behaviour of the two processes is identical; both can be in one of the following states: *neutral* (doing nothing), *trying* to access the resource R, or *critical section* (i.e., accessing the resource R). Both the processes are initially set to neutral state. Once the processes are in neutral states, they can try to enter the critical section. However, there is the following restriction on how to enter the critical section: if the process P_i (for $i \in \{1, 2\}$) is in *trying* state and P_j (for $j \in \{1, 2\}$ and $j \neq i$) is not in the critical section, then only P_i can enter the critical section. Thus, our task is to ensure that both the processes do not simultaneously access the resource R.

(a) Model the above situation using a labelled transition system. Please choose the labelling of states and actions in a sensible way that is relevant to the above context. Also, give an explanation to each of the labels used by you in your model. **(Points: 3)**

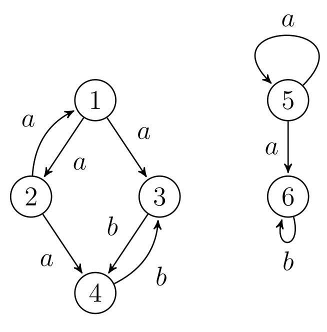
(b) Based on your above model, answer the following queries with explanations.

  i) Does your model guarantees the mutual exclusive access of the resource R? **(Points: 1)**

  ii) Are there any unfair situations in your model, where one process does not allow the other process to enter the critical section forever? How many such unfair situations are present in your model? **(Points: 2)**

Task 2 *Language Equivalence*

Consider the following labelled transition system:



(a) Using the *regular expressions*, give the set of all finite words generated by the states $1, 5$. **(Points: 2)**

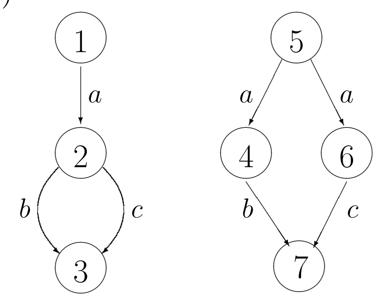
(b) Give the set of all infinite words generated by the states $1, 5$. **(Points: 2)**

(c) Does the states $1, 5$ generate the same set of (in)finite words? **(Points: 2)**

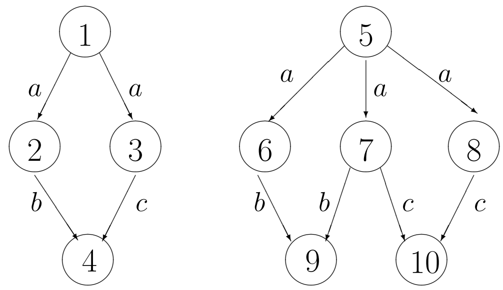
Task 3 *Behavioural equivalences*

Consider the following pair of labelled transition systems with the alphabet $L = \{a, b, c\}$. In each of the pair, establish whether the states 1 and 5 are failure equivalent and bisimilar? Explain your answers and in case of bisimulation, give the relation explicitly. **(Points: 8)**

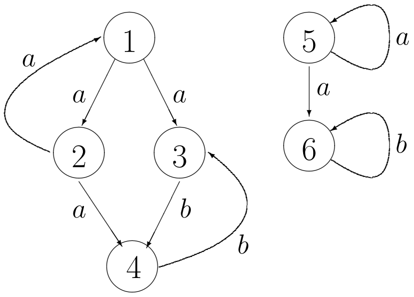
(a)



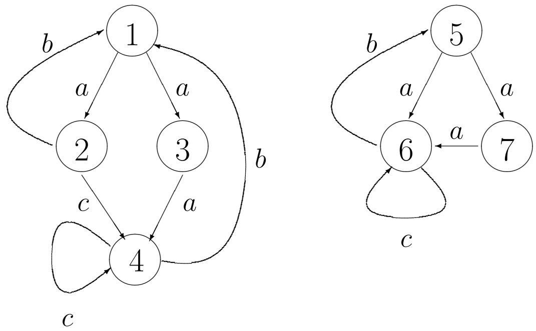
(c)



(b)



(d)



Task 4 *Simulation equivalence*

Let $(S, L, \rightarrow)$ be an labelled transition system. A binary relation $R \subseteq S \times S$ is a *simulation* relation if and only if the following condition is satisfied.

$$\forall_{s,t,s' \in S} \forall_{a \in L} \left((sRt \wedge s \xrightarrow{a} s') \implies \exists_{t' \in S} \ (t \xrightarrow{a} t' \wedge s'Rt') \right).$$

A state t *simulates* the behaviour of another state s, denoted $s \preceq t$, if there exists a simulation relation R such that sRt.

(a) Show that the relation $\preceq$ is a preorder on the set of states S. **(Points: 2)**

(b) Consider the *simulation equivalence* on the set of states S defined in the following way: $s \approx t \iff s \preceq t \wedge t \preceq s$. In other words, two states s and t are simulation equivalent if and only if t simulates the behaviour of s and vice versa.

Does simulation equivalence coincides with bisimilarity, i.e., whether is it the case that for any two states s, t we have s and t are bisimilar if and only if s and t are simulation equivalent? If yes, then prove it. If not, give an example supporting your claim. **(Points: 3)**