Deriving Bisimulation Congruences with Borrowed Contexts[⋆]
(Abstract)

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In the last few years the problem of deriving labelled transitions and bisimulation congruences from unlabelled reaction or rewriting rules has received great attention. This line of research was motivated by the theory of bisimulation congruences for process calculi, such as the π-calculus [19, 14]. A bisimilarity defined on unlabelled reduction rules is usually not a congruence, that is, it is not closed under the operators of the process calculus. Congruence is a desirable property since it allows one to replace a subsystem with an equivalent one without changing the behaviour of the overall system and futhermore helps to make bisimilarity proofs modular.

Previous solutions have been to either require that two processes are related if and only if they are bisimilar under all possible contexts [15] or to derive a labelled transition system manually. Since the first solution needs quantification over all possible contexts, proofs of bisimilarity can be very complicated. In the second solution, proofs tend to be much easier, but it is necessary to show that the labelled variant of the transition system is equivalent to the unlabelled variant.

So the idea which was formulated in the papers of Leifer/Milner [12, 13], Sewell [22] and Sassone/Sobociński [20] is to automatically derive a labelled transition system such that the resulting bisimilarity is a congruence. A central concept of this approach is to formalize the notion of minimal context which enables a process to reduce. Consider, for example, the CCS process $a.P$. It reduces when put into the contexts $\_ \mid \bar{a}.Q$ and $\_ \mid \bar{a}.Q \mid b.R$, but one is interested only in the first context, since it is in some sense smaller than the second one. This yields the labelled transition

$$a.P \xrightarrow{\_|\bar{a}.Q} P \mid Q,$$

saying that $a.P$ put into this contexts reacts and reduces to $P \mid Q$. Using all possible contexts as labels would also result in a (coarser) bisimulation congruence, but we do not gain anything compared to quantification over all contexts (for a more detailed study of this congruence see [3]).

In [12, 13] the notion of "minimal context" is formalized as the categorical concept of relative pushout (RPO) respectively idem pushout (IPO). This notion

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has also been applied to bigraphs [9]. However, the theory is complicated by the fact that one can not work with isomorphism classes of graphs, since in this case the category under consideration would not possess all necessary relative pushouts. Thus one is forced to give unique names to all edges and nodes in a graph, i.e., to add support to a category, and to either work in a precategory or to construct a suitable category starting from such a precategory. A different approach, presented by Sassone and Sobociński [20, 21], that does not require the notion of support, is to construct relative pushouts (so-called GRPOs) in a 2-categorical setting. This work is based on the notion of adhesive categories [11].

We will also use adhesive categories and work with adhesive rewriting systems, which can be seen as a generalization of graph rewriting systems [18], a framework which allows to model dynamic and concurrent systems consisting of interconnected components in a natural and intuitive way. Many process calculi such as the π-calculus [8, 16, 10] and the ambient calculus [7] can be translated into this framework. We are specifically interested in the double-pushout (DPO) approach to rewriting [4, 5]. Adding support, as explained earlier, would be possible in theory, but contradicts the philosophy behind graph rewriting where graphs (or more generally objects) are considered only up to isomorphism. Compared to other approaches, in which the derivation of labels is a somewhat complex task, our approach is rather straightforward and simple.

The approach presented here [6] is motivated by the work of Leifer and Milner and other contributions to this area, but does not directly rely on their theory. Instead we present an uncomplicated way of deriving minimal contexts—we call them borrowed contexts—which smoothly extends the DPO approach and which has a very constructive nature. The only categorical concepts that are needed are pushouts and pullbacks. The main difference to previous approaches is that in our case graphs (more generally: the structures which are being rewritten) are objects and not arrows of the category under consideration. Our arrows instead are (graph) morphisms which provide the necessary tracking information for nodes and edges which, in the case of graphs as arrows, can—as it turned out—only be provided by either adding support to a category or by working in a 2-categorical framework.

Our main result states that bisimilarity defined on labelled transitions with borrowed contexts is indeed a congruence relation. Furthermore we introduce an up-to-context proof technique and discuss the mechanization of bisimulation proofs (see also [17]).

We will compare with related work and present an application of our approach to the derivation of bisimulation congruences for CCS [2]. Finally, we give an outlook to future plans where we are working towards an inductive definition, in SOS style, of the labelled transition system associated to the reduction rules (see also [1]).

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